Geometry of submanifolds with respect to ambient vector fields

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Abstract. Given a Riemannian manifold N^n and $\mathcal{Z} \in \mathfrak{X}(N)$, an isometric immersion $f\colon M^m \to N^n$ is said to have the *constant ratio property with respect to* $\mathcal{Z}$ either if the tangent component $\mathcal{Z}_f^T$ of $\mathcal{Z}$ vanishes identically or if $\mathcal{Z}_f^T$ vanishes nowhere and the ratio $\|\mathcal{Z}_f^\perp\|/\|\mathcal{Z}_f^T\|$ between the lengths of the normal and tangent components of $\mathcal{Z}$ is constant along M^m. It has the *principal direction property with respect to* $\mathcal{Z}$ if $\mathcal{Z}_f^T$ is an eigenvector of all shape operators of f at all points of M^m. In this talk I will report on a joint work with F. Manfio and J. van der Veken, in which we have studied isometric immersions $f\colon M^m \to N^n$ of arbitrary codimension that have either the constant ratio or the principal direction property with respect to distinguished vector fields $\mathcal{Z}$ on space forms, product spaces $\mathbb{S}^n \times \mathbb{R}$ and $\mathbb{H}^n \times \mathbb{R}$, where $\mathbb{S}^n$ and $\mathbb{H}^n$ are the n-dimensional sphere and hyperbolic space, respectively, and, more generally, on warped products $I \times_\rho \mathbb{Q}_\epsilon^n$ of an open interval $I \subset \mathbb{R}$ and a space form $\mathbb{Q}_\epsilon^n$. Starting from the observation that these properties are invariant under conformal changes of the ambient metric, we have provided new characterization and classification results of isometric immersions that satisfy either of those properties, or both of them simultaneously, for several relevant instances of $\mathcal{Z}$ as well as simpler descriptions and proofs of some known ones for particular cases of $\mathcal{Z}$ previously considered by many authors.